Learning unary automata

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30-June-2005, Descriptional Complexity of Formal Systems

Outline

1. Introduction
   - Unary Regular Languages
   - Algorithmic Learning Theory
2. Consistency Problems and PAC Learning
   - Minimum Consistent DFA
   - Minimum Consistent NFA
   - PAC Learning and VC Dimension
3. Learning with Equivalence Queries

Outline

Basics on Unary DFAs.

- A unary language is defined over $\Sigma = \{a\}$.
- A unary regular language is represented by a DFA:

- Ultimate period of a regular language = cycle length.
- Minimum ultimate period = minimum cycle length.
- A unary DFA that consists only of a cycle is called a cyclic DFA.

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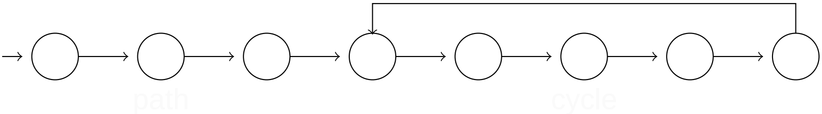

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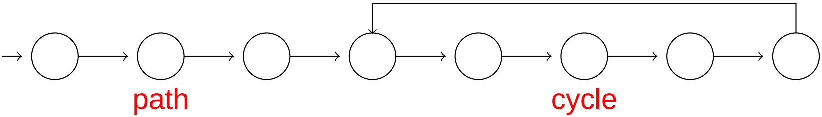

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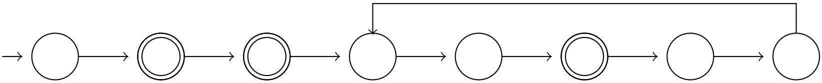

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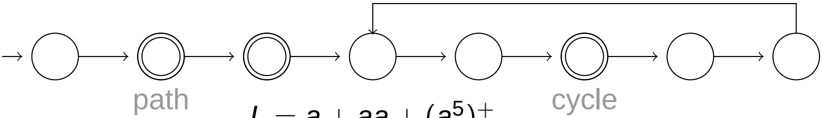

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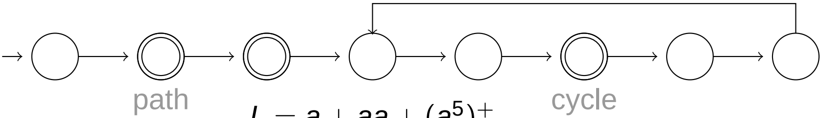

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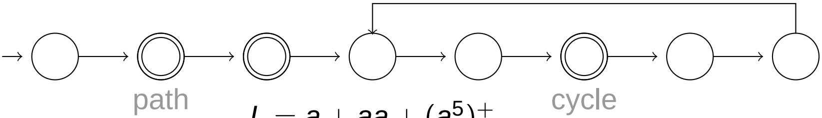

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Basic Notions in Algorithmic Learning Theory

- Concept $c \subseteq X$.
- Concept class $\mathcal{C} \subseteq \mathcal{P}(X)$, concept $c \in \mathcal{C}$.
- Example $x \in X$ is a positive example (for c), if $x \in c$ and a negative example, if $x \notin c$.
- Common problem: Given a set of classified examples, give a good hypothesis for the concept.

In our context:

- Concept class: class of unary regular languages representable by automata of a certain size.
- Concept: language from the concept class.
- Universe X: set of words $\{a\}^*$.
- Example: word.

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- Output: size of a minimum DFA, consistent with P and N.
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- Easy, if we fix cycle length z first.
  - We say example lengths x, y collide modulo z, if $x \equiv y \pmod{z}$
  - If x, y collide modulo z, at least one of them must not reach the cycle in a consistent DFA with cycle length z.
  - Optimum path length = $1+\max\{\min(x, y) \mid x, y$ collide modulo $z\}$.
  - Optimum size = $z+$ optimum path length.
- Compute the size of a minimum consistent DFA.

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Efficiency of the Algorithm

- Number of iterations = optimum size.
- Obvious: optimum size $\leq$ |longest example| + 1.
- Representing inputs: example lengths are coded in binary, input length $\approx \ell = \sum_{x \in P \cup N} \log x$.
- Optimum size $\leq \ell^3$
- Linear time for each iteration (fixed cycle length).
- Total time $O(\ell^4)$.

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A unary NFA with $O(opt^2)$ states can be constructed efficiently.

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A is a PAC (probably approximately correct) algorithm, if

- A determines how many examples it needs in dependence on input parameters $0 < \delta < 1/2$ and $0 < \epsilon < 1/2$.
- Classified examples are chosen randomly under some distribution $\mathcal{D}$ and presented to A.
- With probability $\geq 1 - \delta$, A computes a hypothesis that classifies only an ϵ-portion (measured under $\mathcal{D}$) of examples incorrectly.

Relation to the consistency problem:

- An algorithm that solves the consistency problem for $\mathcal{C}$ on $\Theta(\frac{1}{\epsilon}\log\frac{1}{\delta} + \frac{\mathrm{VC}(\mathcal{C})}{\epsilon}\log\frac{1}{\epsilon})$ examples is a PAC algorithm.

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- An algorithm that solves the consistency problem for $\mathcal{C}$ on $\Theta(\frac{1}{\epsilon}\log\frac{1}{\delta} + \frac{\mathrm{VC}(\mathcal{C})}{\epsilon}\log\frac{1}{\epsilon})$ examples is a PAC algorithm.

PAC Learning

A is a PAC (probably approximately correct) algorithm, if

- A determines how many examples it needs in dependence on input parameters $0 < \delta < 1/2$ and $0 < \epsilon < 1/2$.
- Classified examples are chosen randomly under some distribution $\mathcal{D}$ and presented to A.
- With probability $\geq 1 - \delta$, A computes a hypothesis that classifies only an ϵ-portion (measured under $\mathcal{D}$) of examples incorrectly.

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VC Dimension of a Class of Unary Languages

VC Dimension of $\mathcal{L}_n$

Let $\mathcal{L}_n$ be the class of unary languages, accepted by DFAs with at most n states and $\pi(n)$ be the number of primes $\leq n$. Then

$$n - 1 + \lfloor \log(\pi(n) + 1) \rfloor \;\leq\; \mathrm{VC}(\mathcal{L}_n) \;\leq\; n + \log(n).$$

- Bounds are almost tight: $\log(\pi(n) + 1) \approx \log n - \log \ln n$.

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Outline

Learning with Equivalence Queries

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- Oracle compares the hypothesis with the concept c from $\mathcal{C}$ and
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Angluin, 1990

Non-unary DFAs and NFAs are not learnable from equivalence queries with polynomially many counterexamples and hypotheses of polynomial size.

Learning Cyclic DFAs with Equivalence Queries

Let $\mathcal{C}_n = \mathcal{H}_n$ be the class of languages, that are accepted by a cyclic unary DFA with prime cycle length $\leq n$.

Learning prime cycles with equivalence queries

- There is a learning algorithm that needs at most $O(\frac{n^2}{\ln n})$ counterexamples to learn an arbitrary concept from $\mathcal{C}_n$.
- There is an oracle, such that any learning algorithm needs at least $\Omega(\frac{n^2}{\ln n})$ counterexamples.

Using larger hypotheses

If we allow unary cyclic DFAs with at most n^d $(d \leq n)$ states as hypotheses to learn concepts from $\mathcal{C}_n$, then the upper bound is $O(\frac{n^2}{d})$, whereas the lower bound is $\Omega(\frac{n^2}{d} \cdot \frac{\ln d}{(\ln n)^2})$.

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- Produce hypotheses with prime cycle length p consistent with previous counterexamples.
- Colliding examples modulo p indicate that cycle length p is impossible for the concept.
- At most p counterexamples for each prime p until collision.
- The algorithm needs at most

$$\sum_{p \leq n, p \text{ prime}} p = \Theta\left(\frac{n^2}{\ln n}\right).$$

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- Oracle acts like someone who is cheating in the game of battle ships.
- The concept is not fixed, but constructed depending on the queries.
- Just make sure, the concept is consistent with the counterexamples.
- Every counterexample reveals as little information as possible.
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▸ Skip Example

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Example for the Malicious Oracle

Hypothesis

Examples

	2	3	5	7

List of residues

2	3	5	7
0	0	0	0
1	1	1	1
	2	2	2
		3	3
		4	4
			5
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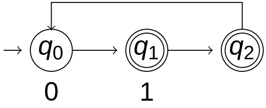

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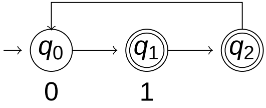

Examples

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140	0	2	0	0

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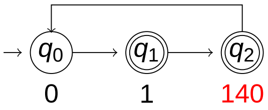

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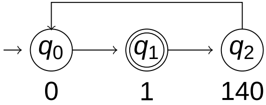

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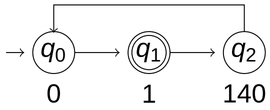

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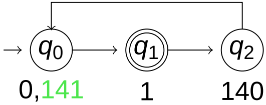

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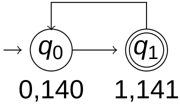

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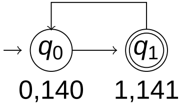

Examples

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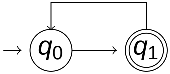

0,140, 1,141

106

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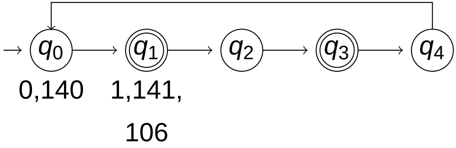

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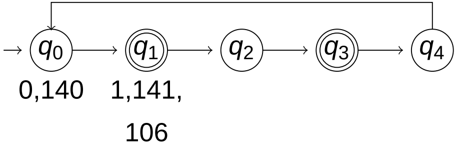

Examples

	2	3	5	7
127	1	1	2	1

List of residues

2	3	5	7
0	0	0	0
1	1	1	1
	2	2	2
		3	3
		4	4
			5
			6

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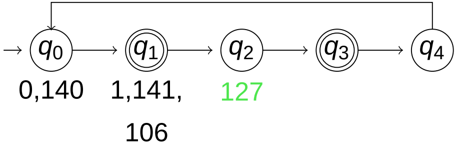

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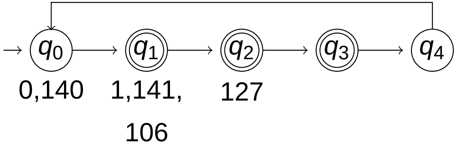

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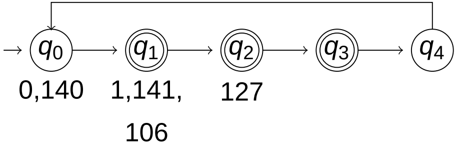

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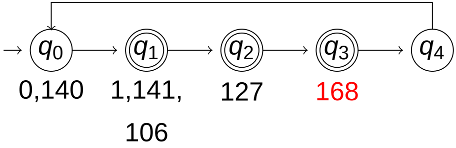

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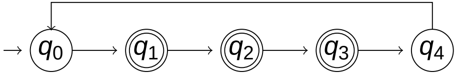

0,140 1,141, 106 127

...
At least $\sum_{p \leq n}(p-2) = \Omega\left(\frac{n^2}{\ln n}\right)$ counterexamples.

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Summary of Our Contributions (I)

Consistency and PAC learning for DFAs

- The consistency problem for unary DFAs becomes simple.
- The VC dimension of unary DFAs with $\leq n$ states is $n + \log n \pm \Theta(\log\log n)$.
- Unary DFAs are efficiently PAC learnable.

Consistency and PAC learning for NFAs

- The consistency problem remains hard for unary NFAs, but can efficiently be approximated quadratically.
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Learning cyclic DFAs by equivalence queries

- Learn DFAs with $p \leq n$ states by hypotheses with $p \leq n$ states: $\Theta\left(\frac{n^2}{\ln n}\right)$ counterexamples are sufficient and necessary.
- Learn DFAs with at most n states by hypotheses with at most n^d states ($d \leq n$): $O(\frac{n^2}{d})$ counterexamples are sufficient and $\Omega(\frac{n^2}{d} \cdot \frac{\ln d}{(\ln n)^2})$ are necessary.

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Thanks for the attention.